in preventing the cathode from melting at extremely high currents. It should be noted that a high degree of ionization is the principal characteristic of the hydrogen propellant in the temperature range involved. Also, the gas is in the neutral plasma state at all times. Since the main species of the frozen flow are protons and electrons, perfect-gas thermodynamic conditions are approached.

The interesting phenomena described open up a new field of possibilities for thermo-ionic acceleration. Table 2 is a forecast of the results obtainable if the heat-transfer efficiency can be increased from 70 to 90% and specific impulses higher than 10,000 sec are attained. This is the scope of the future work.

Space Station Design Parameter Effects on Artificial g Field

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Nomenclature

x, y, z	=	axes attached and rotating with the space station, such that z is the spin axis
A, B, C	=	principal moments of inertia about the x, y, z axes, respectively
$\mathbf{w}$	=	spin vector
p, q, r	=	components of the spin vector about the x, y, z axes, respectively
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	=	unit vectors directed along the x, y, z axes, respectively
0	=	subscript denoting initial value of a parameter
k_1	=	$[(C - B)/A]r_0$
k_2	=	$[(C - A)/B]r_0$
K	=	$(k_1k_2)^{1/2}/r_0$

The main purpose for rotating a manned space station is to provide a simulated gravity field that is habitable by man, i.e., a field whose magnitude and directional changes are kept within the physiological tolerance limits of the man. A man on board a rotating space station may have his vestibular system stimulated in two different ways: 1) by normal head movements in the conduction of his tasks or 2) by disturbed motion of the space station which causes it to rotate simultaneously about two or more of its axes. The purpose of this investigation is to point out the effects imposed by the latter condition on the artificial gravity field.

For a configuration rotating in a passive state (no energy inputs), the gravity field acting on a man positioned along the x axis is given by

$$ng = -\dot{\mathbf{w}} \times \mathbf{r} - \mathbf{w}\,x(\mathbf{w} \times \mathbf{r}) = x[\mathbf{i}(q^2 + r^2) - \mathbf{j}(pq + \dot{r}) - \mathbf{k}(pr - \dot{q})] \quad (1)$$

Equation (1) assumes that energy dissipation can exist in the absence of external torques, and therefore the only stable condition for a rotating system is rotation about the axis of maximum moment of inertia. The disturbed motion of this type of configuration, which has its moments of inertia related by $C > B > A$, is not readily described, because the equations of motion (Euler's dynamic equations) are not directly integrable. Hence, the polhode cone and trace on the inertia ellipsoid which describes this motion[1] must be looked at in more detail. The polhode cone and its trace in the xz plane are shown in Fig. 1.

It is noted that the major factor causing the trace to be distorted in the xz plane is the variance in spin velocity r, which results from the energy coupled from about the other

Received January 2, 1963; revision received July 15, 1963.
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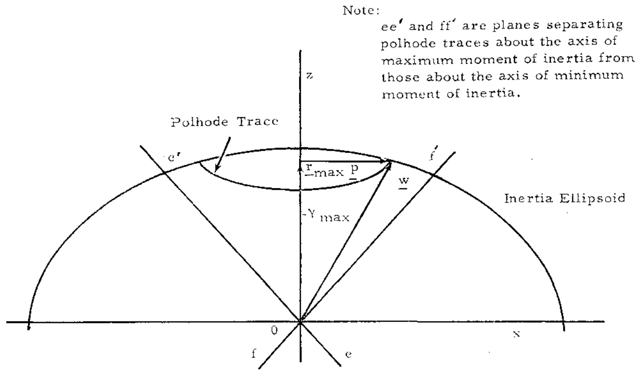

Fig. 1 Polhode trace in the xz plane.

axes. To analyze this variance, the equation of the trace in this plane is examined. It is given by

$$A(A - B)p^2 + C(C - B)r^2 = L^2 \quad (2)$$

From Fig. 1, it is seen that p can be expressed as a function of the spin velocity r by

$$p = r \tan\gamma \quad (3)$$

Then Eq. (2) can be rewritten as

$$A(A - B)r^2 \tan^2\gamma + C(C - B)r^2 = L^2$$

or

$$r = L[C(C - B) - A(B - A) \tan^2\gamma]^{-1/2} \quad (4)$$

Referring to Fig. 1 again, it is noted that, in the motion of the $\mathbf{w}$ vector about the z axis, the maximum angle swept out in the xz plane is γ; hence, this angle is directly related to the size of the polhode cones about the z axis. Using this relationship, the variance in r can be calculated with respect to the mean value of r during a rotation of $\mathbf{w}$ by the following expression:

$$r\% = [(r - r_0)/(r + r_0)]100 \quad (5)$$

where r is the spin velocity at $\gamma_{\max}$ and r_0 the spin velocity at $\gamma = 0$

Considering several types of configurations, a plot of percent variation of r with γ is determined and presented in Fig. 2. For these configurations, it is seen that for small disturbances ($\gamma \leq 7°$) there is less than a 1% variation in the spin velocity r. Hence, for small disturbances the equations of motion for the $C > B > A$ configuration can be written, in general, as

$$\left.\begin{aligned} A\dot{p} + (C - B)qr &= 0 \\ B\dot{q} + (A - C)pr &= 0 \\ r &= r_0 \end{aligned}\right\} \quad (6)$$

The solutions of Eqs. (6) become

$$\begin{aligned} p &= p_0 \cos Kr_0t - (k_1/Kr_0)q_0 \sin Kr_0t \\ q &= q_0 \cos Kr_0t + (k_2/Kr_0)p_0 \sin Kr_0t \\ r &= r_0 \end{aligned} \quad (7)$$

Substituting the values for p, q, and r as derived in Eq. (7) into Eq. (1) gives

$$\begin{aligned} ng = x\{&\mathbf{i}[q_0^2 \cos^2 Kr_0t + \tfrac{1}{2}(k_2/k_1)^{1/2}p_0q_0 \sin 2Kr_0t + (k_2/k_1)p_0^2 \sin^2 Kr_0t + r_0^2] - \mathbf{j}[-\tfrac{1}{2}(k_1/k_2)^{1/2}q_0^2 \sin 2Kr_0t + \\ &p_0q_0(\cos^2 Kr_0t - \sin^2 Kr_0t) + \tfrac{1}{2}(k_2/k_1)^{1/2}p_0^2 \sin 2Kr_0t] - \\ &\mathbf{k}[(1 - k_2/r_0)p_0r_0 \cos Kr_0t - (k_1/k_2)^{1/2}(1 - k_2/r_0)q_0r_0 \sin Kr_0t]\} \end{aligned} \quad (8)$$

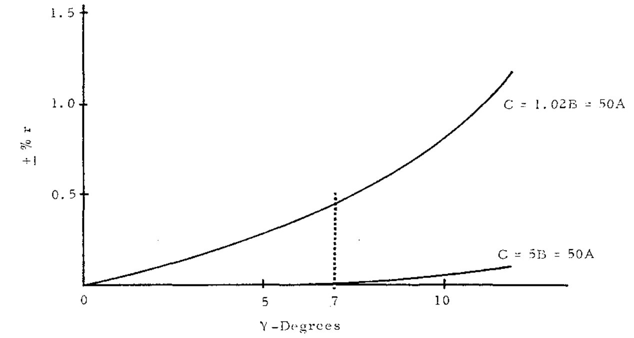

Fig. 2 Spin velocity variation vs γ.

An analysis of Eq. (8) shows that the field along each of the three body axes is oscillatory in direction and magnitude. Along the x axis, man's "down" direction, the resulting acceleration is composed of oscillatory components superimposed on the constant term, xr_0^2. The accelerations perpendicular to the x axis given by the **j** and **k** terms give an interesting result when plotted (see Fig. 3). This figure shows that the gravity vector in the yz plane will vary in a "figure eight" with respect to man's down direction.

The characteristics of the oscillatory acceleration terms in Eq. (8) are directly related, both in magnitude and frequency, to the moment of inertia relationship of the rotating space station. In regard to magnitude, the ratio k_2/k_1 is plotted vs the ratio C/B for various C/A ratios in Fig. 4. In this figure it is noted that for a given C/A value, the factor k_2/k_1 equals one for two different values of C/B and in between these values reduces to a minimum. An initial response to this result may be to select the minimum k_2/k_1 value of a selected C/A curve as defining a best configuration, but it should be noted that the ratio k_1/k_2 is an inverse function and will tend to magnify the magnitudes of the terms it affects with an end result of a greater fluctuation in the field than with $k_2/k_1 = 1$. Since it is desirable to reduce all acceleration component magnitudes simultaneously in order to reduce adverse effects to the man,[2] it is felt that $k_2/k_1 = 1 \pm 10\%$ should be the range selected initially for the moment-of-inertia relationship of a given configuration. The parameter K, which affects the oscillation frequency, increases in magnitude from the lower to higher value of C/B for a given C/A value, indicating that the period of oscillation is longer for a dumbbell than for a wheel configuration.

From the foregoing discussion, it is found that the constant magnitude and direction centrifugal force field normally associated with constant rotation about an axis transforms into a field of complex oscillatory magnitude and frequency characteristics when a space station exhibits disturbed rotational motion. In general, it was shown that these characteristics were dependent on the moment-of-inertia relationship for a given configuration. It is apparent that many other contingencies could be pursued but are beyond the scope of this report. Therefore, it is hoped that the analysis presented will at least serve to enlighten space station designers with the fact that man's physiological tolerance limits, especially those related to acceleration and frequency changes, must be carefully considered in the selection of artificial gravity space station design parameters.

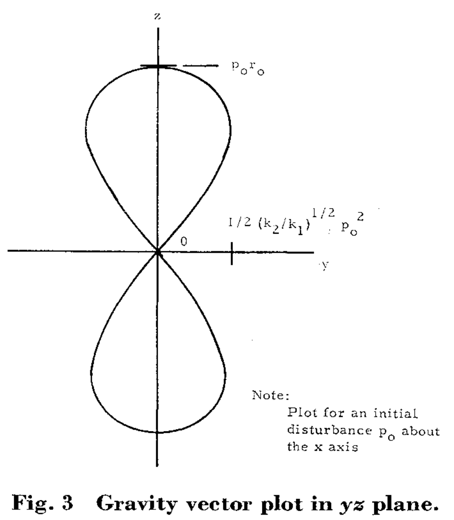

Fig. 3 Gravity vector plot in yz plane.

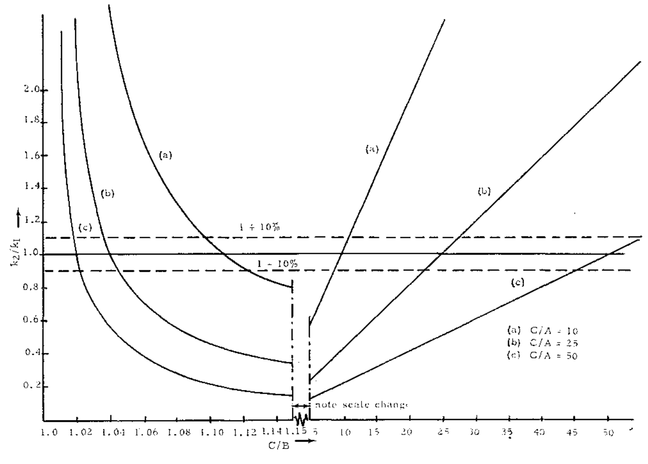

Fig. 4 k_2/k_1 vs C/B for various C/A values.

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Umbra and Penumbra Eclipse Factors for Satellite Orbits

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A. Introduction

THE amount of time a satellite is occulted by the earth shadow during an orbit is of major consequence in determining the satellite thermal control system, the power supply (if powered by a solar source), and the atmospheric control system. To a lesser extent, the satellite external torque history and the sensor systems are also influenced by the time the satellite spends in the earth shadow.

B. Physics of the Problem

The earth shadow consists of two regions, the umbra and the penumbra, as shown in Fig. 1. The umbra is the conical total shadow projected from the earth on the side opposite the sun. In this region, the solar radiation intensity is zero. The penumbra is the partial shadow between the umbra and the full-light region. In the penumbra, the light of the sun is only

Received December 23, 1963; revision received April 28, 1964.

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